LITERATURE CITED

1. M. A. Berliner, Humidity Measurement [in Russian], Moscow (1973).
2. Yu. G. Zav'yalov, A. M. Litvinov, and V. P. Mishta, "Mathematical description of operation of a heated lithium chloride humidity converter", Autometry, Vol. 2 [in Russian], Novosibirsk (1981).
3. D. Kheintse, Izmirit. Tekhn., No. 9, 51-54 (1982).
4. S. P. Khedlin and F. N. Trofimenkov, Humidity, Vol. 1, Principles and Methods of Humidity Measurement in Gases [in Russian], Leningrad (1967), pp. 225-234.
5. V. V. Nikonova and S. M. Kochetov, Pr. of the Hydrometeorology Device Scientific Research Institute, No. 35, [in Russian], Moscow (1978), pp. 86-90.
6. A. N. Tikhonov and A. A. Samarskii, Equations of Mathematical Physics [in Russian], Moscow (1973).
7. M. Abramovits and I. Tigan (eds.), Special Functions [Russian translation], Moscow (1979).
8. L. V. Kantorovich and V. I. Krylov, Approximate Methods of Higher Analysis [in Russian], Moscow (1962).
9. T. L. Ibbs and K. E. Gryu, Thermal Diffusion in Gases [in Russian], Moscow (1956).
10. E. N. Meison and L. Monchik, Humidity, Vol. III, Physical Principles, Basic Definitions, and Control Devices [in Russian], Leningrad (1969), pp. 310-335.

MOTION OF A GRANULAR MATERIAL OVER A FIXED PLANE

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It is shown that there exists an excess viscous friction produced by shear flow of a continuous medium in the gap between the moving particle layer and the slip plane.

Introduction. In the motion of a granular material on a fixed plane the Coulomb friction law is not always obeyed. This occurs to an extreme degree in the motion of thin layers of powdered materials. It is logical to propose the existence of additional viscous forces caused by shear flow of a continuous medium in the gap between the moving particle layer and the slip plane. This study will evaluate this force.

According to the physical model to be used, we will represent the tangent stresses on the slip surface in the form of a sum of Coulomb and viscous friction

$$\tau = \sigma f + \mu \frac{v}{h}. \tag{1}$$

Neglecting the friction between the outer surface of the material layer and the continuous medium and assuming constancy of particle velocity over layer thickness, which is fully justified for thin layers where the coefficient of external friction of the material is less than the coefficient of internal friction, we write the equation of conservation of momentum in projections in the slip plane, inclined to the horizontal at an angle α:

$$\rho\delta \frac{dv}{dt} = \rho\delta g \sin\alpha - \rho g \delta \cos\alpha\, f - \mu \frac{v}{h}. \tag{2}$$

The layer thickness can be found from the equation of conservation of mass

$$Q = v\delta B\rho. \tag{3}$$

It follows from Eqs. (2) and (3) that

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TABLE 1. Experimental and Calculated Characteristics

Material	ρ_m, kg/m³	ρ, kg/m³	$d\cdot10^6$, m	f_0	f	$h\cdot10^6$, m	Correlation coefficient	Maximum deviation, %
Nickel hydrocarbonate (specimen 1)	3975	340,2	38,5	0,675	0,673	9,55	0,958	20
Nickel hydrocarbonate (specimen 2)	3975	340,3	38,5	0,675	0,621	11,55	0,924	20
Nickel oxide (specimen 1)	5671	437,1	17,3	0,755	0,755	28,2	0,984	15
Nickel oxide (specimen 2)	5671	430,1	17,3	0,766	0,569	27,9	0,989	11
Beach sand	2270	1172,7	168,5	0,755	0,639	9,38	0,890	21
Cement	3150	1119,6	88,7	0,681	0,627	4,93	0,960	15

$$\frac{dv}{dt} \equiv v\frac{dv}{dx} = g(\sin\alpha - f\cos\alpha) - \frac{\mu B}{Qh}v^2. \tag{4}$$

After integrating Eq. (4) with the conditions t = 0, x = 0, v = v_0, and t = t, x = x, v = v we find:

$$x = -\frac{1}{2A}\ln\frac{c - v^2}{c - v_0^2}; \tag{5}$$

$$t = \frac{1}{2A\sqrt{c}}\ln\frac{(\sqrt{c} + v)(\sqrt{c} - v_0)}{(\sqrt{c} - v)(\sqrt{c} + v_0)}; \tag{6}$$

$$v = (c - (c - v_0^2)\exp(-2Ax))^{0.5}, \tag{7}$$

where

$$A = \frac{\mu B}{Qh}; \quad c = \frac{M}{A}; \quad M = g(\sin\alpha - f\cos\alpha).$$

It can easily be shown that as A → 0 (a condition corresponding to increase in particle dimensions and layer thickness or decrease in viscosity of the continuous medium) Eq. (7) yields

$$v = (2g(\sin\alpha - f\cos\alpha)x + v_0^2)^{0.5}. \tag{8}$$

This equation of equiaccelerated motion corresponds to a purely Coulomb friction law (see, for example, [1-3]). On the other hand, for A > 0, with increase in x the slip velocity tends to the finite value

$$v = \left(\frac{g(\sin\alpha - f\cos\alpha)Qh}{\mu B}\right)^{0.5}. \tag{9}$$

An experimental verification of Eqs. (5)-(7) was carried out in an air medium on a stainless steel trough. The trough had a width of 0.25 m and length of 1.048 or 2.023 m. The pitch of the trough was varied over the range 30-55°. Mass flow rate varied from 0.0126 to 0.05 kg/sec. All measurements were performed in a steady state regime with medium temperature of 30°C. The material velocity was determined by the section method from the expression

$$v = \frac{lQ}{m}. \tag{10}$$

Change in the initial velocity of the material led to acceleration or braking of the layer, although it had no effect on particle velocity in the steady state segment.

The static friction coefficient f_0 was defined as the tangent of the trough slope angle at which material motion was first detected, while the dynamic friction coefficient was determined by processing experimental data on v, α, and Q. In the final reckoning f values were chosen such that the experimental data fit a straight line passing through the origin with coordinates v^2 and Q(sin α−f cos α) with minimum scattering.

To evaluate particle velocity changes over layer thickness experiments were performed on a plexiglass trough. Visual measurements of particle velocity at the upper and lower surfaces revealed that those velocities did not differ over the range of experimental parameters used.

The experimental data were processed by the method of least squares to obtain a linear regression. Final results are presented in Table 1.

The quantity h was calculated from Eq. (9), and Table 1 shows average values. The deviation of the experimental data for h are relative to its mean value.

The results obtained confirm the significant effect of viscous friction, caused by shear flow of the continuous medium in the gap between the moving particle layer and the slip plane.

It can easily be foreseen that in the case where the tangent stresses defined by Eq. (1) exceed the internal friction in the layer, i.e., $\tau > \sigma f_B$, a shear flow develops between the material particles. The assertion made in [4] that that is possible only when $f > d_B$ is a special case, valid for the situation $\sigma f \gg \mu v f h$.

Conclusion. The viscous friction effect observed herein permits a correct approach toward understanding of a number of engineering problems, in particular, description of the flow of powdered materials along the inclined surfaces of thermal furnaces.

NOTATION

σ, normal stress, referenced to slip plane; f, external friction coefficient; μ, dynamic viscosity; v, velocity of particle adjacent to slip plane; h, characteristic thickness of gap between particle layer and plane, the value of which is probably determined by particle size and packing density; ρ, poured density of granular material; δ, layer thickness; g, acceleration of gravity; Q, mass flow rate of material; B, layer width; ℓ, distance between sections; m, mass of material between sections; f_i, internal friction coefficient; ρ_m, density of particle material; d, mean surface particle size; f_0, external static friction coefficient.

LITERATURE CITED

1. D. O. Bytev, A. I. Zaitsev, Yu. I. Makarov, and V. A. Severtsev, Izv. Vyssh. Uchebn. Zaved., Khim. Khimich. Tekh., 23, No. 11, 1437-1441 (1980).
2. O. D. Neikov and N. N. Logachev, Aspiration in Production of Powdered Materials [in Russian], Moscow (1973).
3. I. V. Domanskii, V. P. Isakov, G. M. Ostrovskii, et al., in: Chemical Production Machines and Apparatus, Examples and Problems [in Russian], V. N. Sokolov (ed.), Leningrad (1982).
4. M. A. Goldshtik and B. N. Kozlov, Zh. Prikl. Mekh. Tekh. Fiz., No. 4, 67-77 (1973).